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## TETHER ASSISTED NEAR EARTH OBJECT (NEO) DIVERSION

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Potential earth impact threats by asteroids have motivated researchers to find effective NEO diversion techniques. Several means to perturb the motion of an asteroid have been proposed in the literature. One of the effective non-nuclear techniques is the attachment of a tether and ballast to the asteroid to alter its trajectory. In this paper it is shown that cutting the tether at an appropriate time can enhance the diversion of the asteroid. The instant of cutting the tether significantly affects the final orbit of the asteroid and thus the resulting diversion. It is shown that by cutting the asteroid at proper time a larger diversion can be achieved in a shorter time.

### I. INTRODUCTION

More than 2000 near earth objects (NEO) with diameters greater than 100 m have been discovered by astronomers so far. Asteroids that can get closer than 0.05 AU to the earth, and have diameters larger than 150 m, are considered as potentially hazardous asteroids. There are currently 1075 known potentially hazardous asteroids\*. Potential earth impact threats by asteroids or comets have generated interest in the concept of NEO diversion. The impact of any asteroid or comet with the earth could have a catastrophic effect on life on earth.

An asteroid can be deflected from the earth-crossing orbit by perturbing its motion. Several methods have been proposed in the literature to perturb the motion of an earth crossing asteroid.

Nuclear devices have been considered for asteroid diversion. Nuclear explosions can perturb the asteroid motion in two ways: either through the influence of radiation or by the impulse induced by the ejected parts from the asteroid surface<sup>1</sup>. However, serious political and environmental issues can arise from the use of such devices. To overcome these limitations, several non-nuclear remedies have been proposed.

Use of solar sails instead of nuclear explosions has been suggested by Melosh and Nemchinov<sup>2</sup>. This method is based on focusing sunlight onto the surface of the asteroid to generate thrust by vaporizing the asteroid's surface layers. The advantage of this method is that it does not require any equipment to be landed on the asteroid.

Use of the gravitational coupling (gravity tug) is another technique for asteroid deflection. In this

technique a spacecraft hovers near the asteroid with a continuous low thrust propulsion, in a static relative equilibrium, which causes an acceleration of the centre of mass of the spacecraft-NEO system<sup>3,4</sup>. Gravity is used as a towline in this method.

Another option is firing a large mass to strike the asteroid to alter its kinetic energy and thus its trajectory. It can be shown that small asteroids (10<sup>2</sup> m diameter) can be deflected using this technique by impact of a spacecraft borne mass. Alternatively, a mass driver system can be attached to the asteroid to eject the asteroid material and hence alter its trajectory. This is a long term strategy. Three decades is considered to be a reasonable lead time for this method<sup>1</sup>.

Attachment of a tether and ballast mass to the NEO can also alter the asteroid trajectory in two ways: changing the centre of mass and through the tension induced by the tether. The effect of attaching a long tether and ballast mass to the asteroid has been investigated by French and Mazzoleni<sup>5</sup>. They have shown that by attachment of a long tether and ballast mass to the asteroid, reasonable deflections can be achieved after several years. The advantage of this technique is that the entire asteroid can be deflected rather than fractured by a direct impact because the fracture could cause unpredictable consequences. Additionally, using this technique, it is possible to impart twice the momentum as a direct inelastic impact<sup>6</sup>.

This paper also focuses on the tether assisted asteroid diversion. It has been shown in Reference 6 that the attachment of a long tether and ballast mass can change the orbit of an Earth-threatening asteroid, but the procedure takes several years. In this paper we investigate the effect of attachment of the tether and

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\* <http://neo.jpl.nasa.gov/neo/groups.html/>

ballast mass, and then cutting the tether at proper time, to enhance the diversion achieved, and improve the lead time.

## II. DYNAMICAL MODEL

To start with, a dynamical model of the system needs to be developed. The system considered consists of an asteroid connected to the ballast by a tether. The asteroid and the ballast are modelled as point masses which imply that the effect of rotation of the asteroid has been neglected in this preliminary model. Figure 1 shows the geometrical configuration of the dynamical model.

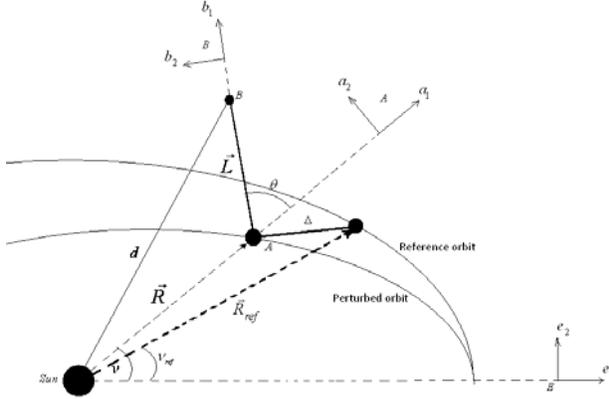


Fig. 1: Geometrical configuration of the asteroid-tether-ballast mass system (before payload release).

In Fig. 1,  $\nu_{ref}$  and  $\vec{R}_{ref}$  are the true anomaly and position vector of the asteroid in the reference orbit, respectively, while  $\nu$  and  $\vec{R}$  are the true anomaly and position vector of the asteroid in the perturbed orbit, respectively. It should be noted that only planar motion is considered in this paper. The tether length and orientation are shown by vector  $\vec{L}$ .  $\theta$  is the inclination of the tether measured from the local vertical coordinate of the asteroid in its perturbed position. Mitigation distance is shown by  $\Delta$ , which is the distance between the position of the asteroid in the perturbed orbit and in the reference orbit. In this paper the mass of the asteroid and the ballast mass are defined by  $m_a$  and  $m_b$ , respectively.

There are three reference frames shown in Fig. 1. Reference frame  $E$  is the fixed reference frame attached to the sun.  $A$  is the LVLH reference frame attached to the asteroid in its perturbed position.  $B$  is the reference frame attached to the tether with unit vector  $\vec{b}_1$  along the tether.

Dynamic analysis of the system starts with deriving its kinetic and potential energies, as follows:

$$K = (m/2)(\dot{R}^2 + R^2\dot{\nu}^2) + m_b L(\dot{\nu} + \dot{\theta})(L/2)(\dot{\nu} + \dot{\theta}) \quad [1]$$

$$+ R\dot{\nu} \cos \theta - \dot{R} \sin \theta]$$

$$V = -GM \{(m_a/R) + (m_b/d)\} \quad [2]$$

For the sake of brevity, the following variable has been defined, which has been shown in Fig. 1.

$$d^2 = R^2 + L^2 + 2RL \cos \theta \quad [3]$$

Once the kinetic and potential energy expressions have been obtained, the equations of motion of the system can be derived using the Lagrange equation:

$$\frac{d}{dt} \left( \frac{\partial K}{\partial \dot{q}_i} \right) - \frac{\partial K}{\partial q_i} + \frac{\partial V}{\partial q_i} = Q_i \quad [4]$$

where  $q_i \equiv R, \nu, \theta$ , and  $Q_i$  are the generalized forces. The equations of motion of the system obtained by Lagrange equation can be written, after some algebraic simplifications and decoupling, as:

$$\ddot{R} = R\dot{\nu}^2 - GM/R^2 + \tilde{m} \cos \theta / (1 + \tilde{m}) \{ -GM/d^3 + \cos \theta (GM/R^2) + L(\dot{\nu} + \dot{\theta})^2 - L(GM/d^3) \} \quad [5]$$

$$\ddot{\nu} = -2\dot{R}\dot{\nu}/R + \tilde{m} \sin \theta / (1 + \tilde{m}) \{ (L/R)(\dot{\nu} + \dot{\theta})^2 + \cos \theta (GM/R^3) - GM \cos \theta / d^3 - GM(L/R)/d^3 \} \quad [6]$$

$$\ddot{\theta} = (1/L) \{ R(GM/d^3) - (GM/R^2) \} \sin \theta + 2(\dot{R}\dot{\nu}/R) - (\tilde{m}/(1 + \tilde{m})) \sin \theta \{ (L/R)(\dot{\nu} + \dot{\theta})^2 + (GM/R^3) \cos \theta - \cos \theta (GM/d^3) - (L/R)(GM/d^3) \} \quad [7]$$

Where  $\tilde{m}$  is the mass ration defined as:

$$\tilde{m} = m_b / m_a \quad [8]$$

In this paper, non-dimensionalized parameters have been used for the analysis. The non-dimensionalized parameters are derived via dividing all length parameters by the semi-major axes of the earth's orbit ( $a_e$ ), and dividing all time rates by the earth mean orbital rate,  $n = (GM/a_e^3)^{1/2}$ .

After applying the binomial expansion, and using non-dimensionalized parameters Equations (5), (6) and (7) can be written as:

$$\begin{aligned} \ddot{R} = & \hat{R}\dot{\nu}^2 - 1/\hat{R}^2 + (\tilde{m}/(1 + \tilde{m})) \cos \theta \{ (L/\hat{R}^3 a_e) \} (3 \cos^2 \theta \\ & - 1) + (L/\hat{R}^2 a_e)^2 \cos \theta (3/2)(3 - 5 \cos^2 \theta) \\ & + (L/a_e)^3 (\nu' + \theta')^2 \} \end{aligned} \quad [9]$$

$$v'' = -2(\widehat{R}'v'/\widehat{R}) + (\widehat{m}/(1+\widehat{m}))\sin\theta\{(L/(\widehat{R}a_e))(v' + \theta')^2\} \quad [10]$$

$$+ (L/a_e)^2(\cos\theta/\widehat{R}^5)(3/2)(3-5\cos^2\theta)$$

$$+ (L/a_e)((3\cos^2\theta-1)/\widehat{R}^4)\} \quad [11]$$

$$\theta'' = \{-3\cos\theta/\widehat{R}^3 + (L/(\widehat{R}^4a_e))(3/2)(-1+5\cos^2\theta)\}\sin\theta$$

$$+ 2(\widehat{R}'v'/\widehat{R}) - (\widehat{m}/(1+\widehat{m}))\sin\theta\{(L/(\widehat{R}a_e))(v' + \theta')^2\}$$

$$+ (L/a_e)^2(\cos\theta/\widehat{R}^5)(4.5-7.5\cos^2\theta) + (L/a_e)(3\cos^2\theta-1)/\widehat{R}^4\}$$

Where  $\widehat{R}$  is the non-dimensionalized distance between the sun and the asteroid. Prime is used to indicate derivation with respect to  $\tau$  instead of dot which stands for derivation with respect to  $t$ . The following equation shows the relation between derivation with respect to  $t$  and  $\tau$ :

$$\frac{d}{dt} = n \frac{d}{d\tau} \quad [12]$$

Equations (9), (10) and (11) can be solved numerically (e.g using Runge-Kutta method) provided six initial conditions are given.

The diversion of the asteroid from its original path, defined by  $\Delta$  in Fig. 1, can be easily obtained from the following expression:

$$\Delta = (a_e/R_e)((\widehat{R}_{ref}(v_{ref}))^2 + (\widehat{R}(v))^2) \quad [13]$$

$$- 2\widehat{R}_{ref}(v_{ref})\widehat{R}(v)\cos(v - v_{ref}))^{1/2}$$

Equation (13) gives the diversion achieved by attachment of the tether and ballast. Now, assume that at  $v = v_r$  the tether has been cut. After cutting the tether, the asteroid would continue its motion in a new orbit (called final orbit in Fig. 2). The characteristics of the final orbit are determined using the non-dimensionalized velocity of the asteroid at the cutting point,  $\widehat{V}_{cut}$ , true anomaly of the asteroid at the cutting point,  $v_r$ , and its derivative,  $v'_r$ , as follows:

$$a_{new} = a_e \widehat{R}_{cut} / (2 - \widehat{R}_{cut} \widehat{V}_{cut}^2) \quad [14]$$

$$e_{new} = [(\widehat{R}_{cut} \widehat{V}_{cut}^2 - 1)^2 \cos^2(\beta) + \sin^2(\beta)]^{1/2} \quad [15]$$

$$v_i = \cos^{-1}\{(1/e_{new})(\widehat{R}_{cut} \widehat{V}_{cut}^2 \cos^2(\beta) - 1)\} \quad [16]$$

$$\widehat{V}_{cut} = [\widehat{R}_{cut}'^2 + (\widehat{R}_{cut} v'_r)^2]^{1/2} \quad [17]$$

$$\beta = \sin^{-1}(\widehat{R}_{cut}' / \widehat{V}_{cut}) \quad [18]$$

The variation of  $\Delta$  with the system parameters is examined. It is noted that, to obtain a reasonable value of  $\Delta$  within a few years, a very long tether must be used.

### III. SIMULATION RESULTS

The effect of attachment and then cutting the tether is investigated in this section via simulation results. As shown in Fig. 2, cutting the tether will change the asteroid orbit. The instant of which the tether is cut, has a significant effect on the final orbit of the asteroid.

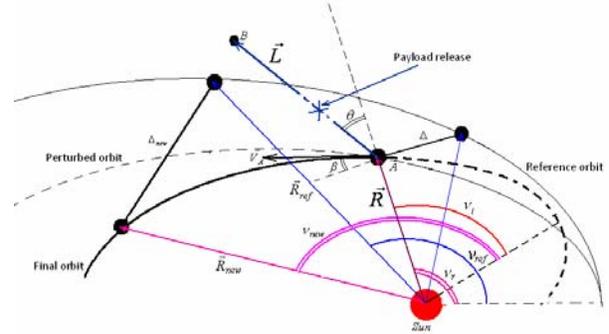


Fig. 2: Configuration of the Asteroid-Tether-ballast mass system (including payload release).

The dashed ellipse in Fig. 2 shows the final orbit of the asteroid, extrapolated to show the perihelion of the final orbit.

Figures 3 and 4 show a comparison of  $\Delta_{cut}$  ( $\Delta$  with tether cut) and  $\Delta$ .

The objective is to achieve a sufficiently larger diversion in a shorter time by selecting the point of tether severance judiciously. Table I shows the parameters used for simulation results presented in Figs. 3 and 4.

$e$	0.8
$a$	1.2 (AU)
$v(0)$	0 (rad)
$\theta'(0)$	0 (rad / ( $\frac{1}{n}$ s))
$\theta(0)$	0 (rad)

Table I: Baseline parameters for simulation results in Figs. 3 and 4.

The length of the tether is  $L = 10000$  Km and the mass ratio is  $\widehat{m} = 0.001$ .

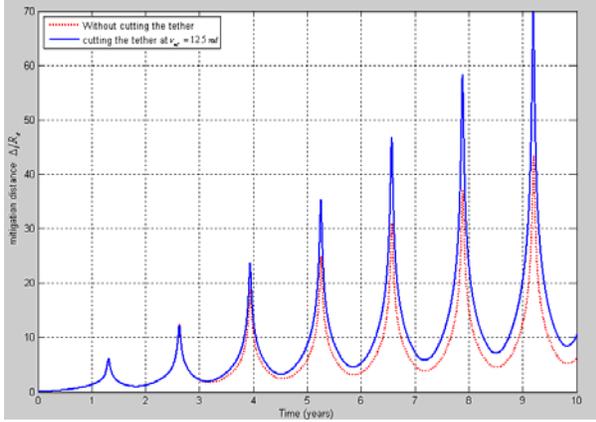


Fig. 3: comparison of the mitigation distance with and without cutting the tether, versus time.

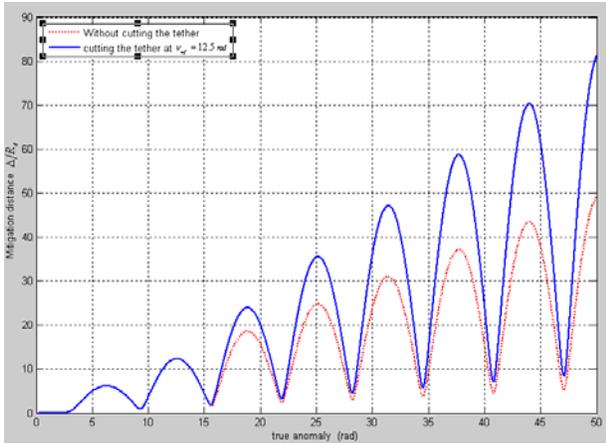


Fig. 4: comparison of the mitigation distance with and without cutting the tether, versus true anomaly.

Although Figs. 3 and 4 show the diversion of the asteroid from its original path, they do not show the distance from the earth orbit, which is the main objective. In this section analysis has been conducted to examine the diversion from the earth orbit achieved by tether attachment and by cutting the tether some time after attachment.

**Example I**

For the first example, the baseline parameters have been chosen to be close to those of "99942 Apophis", as shown in Table II.

$e$	0.191211
$a$	0.9224193 (AU)
$\lambda$	0.8825 (rad)
$v(0)$	0 (rad)
$\theta(0)$	0 (rad)
$\theta'(0)$	4 (rad / (1/n s))

Table II: Parameters of Apophis

Since the asteroid considered in this example has a low eccentricity, a very long tether is required to obtain sufficient diversion. For this example, tether length is equal to  $L = 10000 \text{ Km}$  and the mass ratio is  $\tilde{m} = 0.003$ .

Figure 5 shows the orbit configuration of the asteroid considered in this example and the earth orbit configuration.

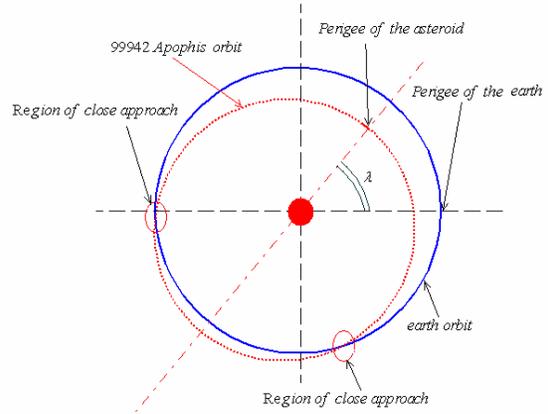


Fig. 5: Orbit configuration for example one.

Figure 6 shows the diversion of the asteroid from its original path versus the reference true anomaly and time, respectively.

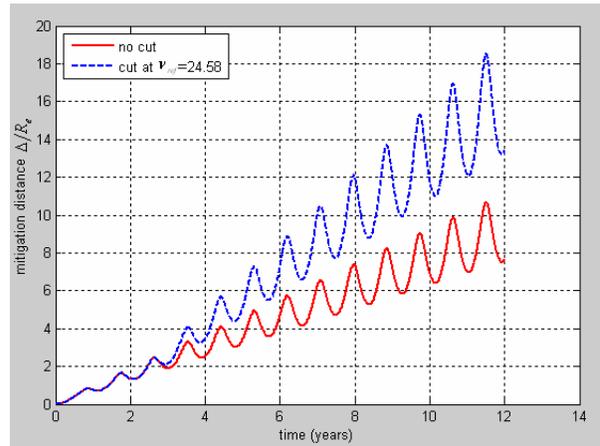


Fig. 6: mitigation distance obtained by attaching the tether, versus time.

It is also interesting to look at the motion of the tether, after attachment to the asteroid. Figure 7 shows the libration angle of the tether and its time derivative, respectively, versus reference orbit true anomaly.

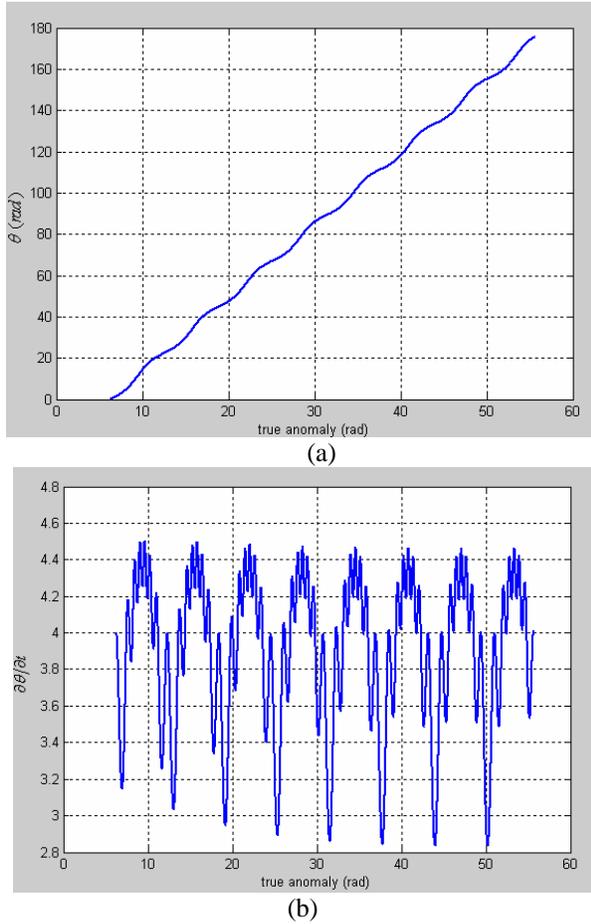


Fig. 7: (a) rotation angle of the asteroid ( $\theta$ ) versus  $v_{ref}$ , (b)  $\partial\theta/\partial t$  versus  $v_{ref}$

Figure 7 illustrates that tether fully rotates around the asteroid. It is worth mentioning that, in this case, the initial tether rotational speed ( $\dot{\theta}(0)$ ), makes tether fully rotate around the asteroid. Without this initial rotational speed, the tether would only oscillate around  $\theta = 0$ , which has an adverse effect on the diversion achieved.

In order to study the effectiveness of the method in preventing catastrophic earth-asteroid collision, the distance between the asteroid and earth should be examined, as shown in Fig. 8.

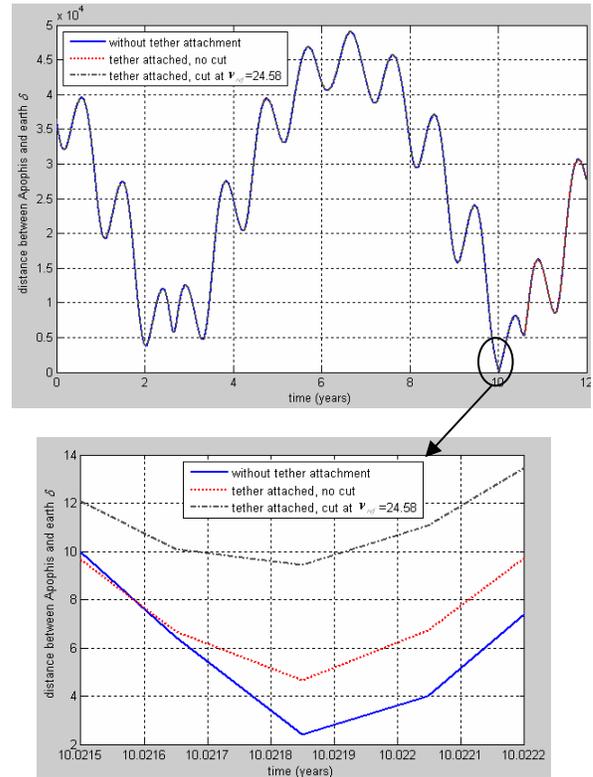


Fig. 8: distance between the asteroid and the earth.

Figure 8, illustrates that in the absence of any mitigating action, the asteroid would approach the earth, as close as 2.25 earth radius. By attaching the tether to the asteroid, this close approach can only be increased to 4.8 earth radiuses, while by cutting the tether at  $v_{ref} = 24.58 \text{ rad}$  it can be increased to 9.5 earth radiuses.

It is also worth mentioning that simulation results have shown points with high rotational rate ( $\theta'$ ), with  $\theta \approx \pi$  are best points to cut the tether.

It should be noted that, sufficient diversion in this example is achieved using a tether length of 10000 Km and a mass ration of 0.003, which are far beyond the current capability of space missions. However, as the asteroid considered in this example has low eccentricity, such values are necessary.

### Example II

For the second example, the parameters of the asteroid have been chosen to be close to those of "2009 KD3". It should be mentioned that, the original parameters have been slightly modified for the sake of simplicity (e.g the small inclination angle has been

ignored and  $t_p$  has been slightly modified). Table III shows the baseline parameters used for simulation.

$e$	0.67309
$a$	2.1528 (AU)
$\lambda$	2.5342 (rad)
$\nu(0)$	0 (rad)
$\theta(0)$	0 (rad)
$\theta'(0)$	0 (rad / ( $\frac{1}{n} s$ ))

Table III: Parameters of KD3.

Because of the higher eccentricity of KD3 compared to Apophis, a shorter tether can be used to obtain sufficient diversion. In this example the tether length is  $L = 3000 \text{ Km}$  and the mass ratio is  $\tilde{m} = .003$ . Figure 9 shows the asteroid orbit configuration and the earth orbit configuration.

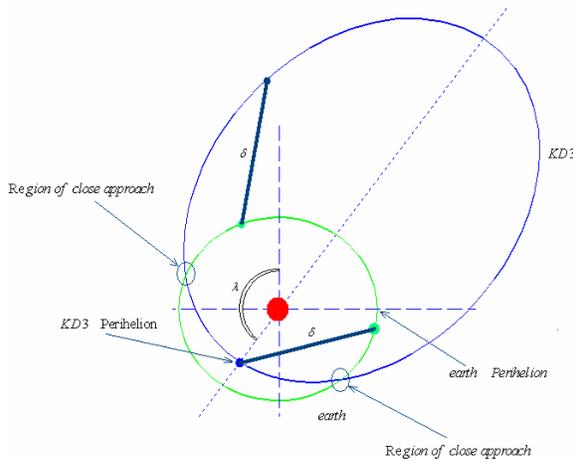
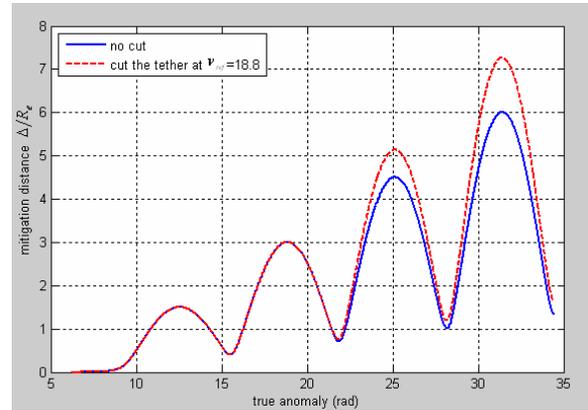


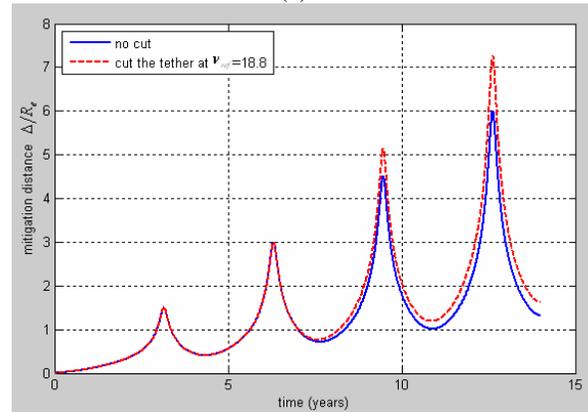
Fig. 9: Asteroid orbit configuration (Example II).mass system (including payload release).

Figure 10 shows the diversion achieved by attachment of the tether and cutting the tether at proper time.

As Fig. 10 illustrates, in this case, cutting the tether at proper time, can only enhance the diversion achieved by 1 earth radius, after around 10-15 years.



(a)



(b)

Fig. 10: (a) rotation angle of the asteroid ( $\theta$ ) versus  $\nu_{ref}$ , (b)  $\partial\theta/\partial t$  versus  $\nu_{ref}$

Based on the asteroid orbit configuration with respect to the earth, the distance between the earth and the asteroid versus time, can be obtained, as shown in Fig. 11.

Figure 11 shows the distance between the earth and the asteroid versus time. The time at the beginning of the simulation is assumed to be zero. Fig. 11 illustrates that in the absence of a mitigating scheme, the Earth and the asteroid would have a very close approach after 12.5 years.

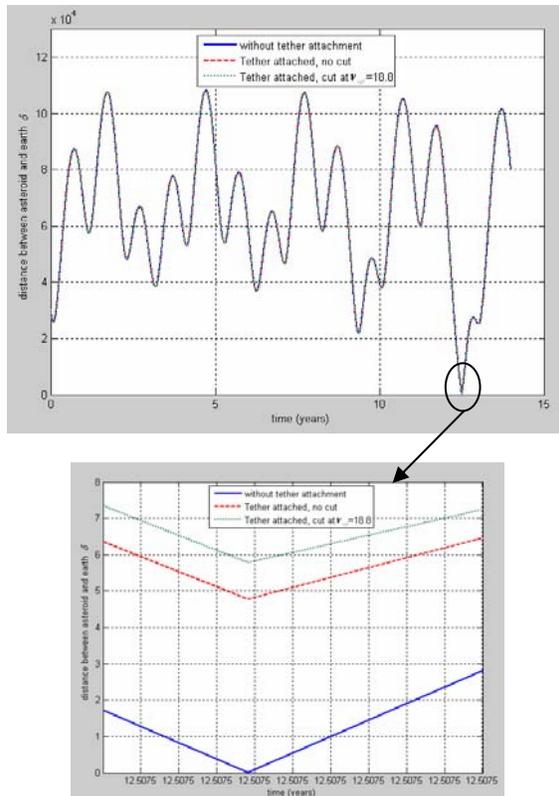


Fig. 11: Distance between the asteroid and the earth (Example II).

Figure 11 reveals the mitigating effect of attachment of a tether and ballast mass to the asteroid, on the close approach distance. It also shows the enhancement achieved by cutting the tether at proper time. While attachment of the tether can increase the close approach distance by 5 earth radius, cutting the tether at proper time can only enhance that by one earth radius.

#### IV. CONCLUSION

In this paper, the effect of attaching a tether and ballast to an asteroid, and then cutting the tether at proper time, on the trajectory of the asteroid has been studied. To examine the effectiveness of the proposed technique, two examples have been discussed; one with low eccentricity and one with high eccentricity. It has been shown via simulation that, attachment of a tether and ballast can alter the trajectory of asteroids with high eccentricity much more than those with low eccentricity. For low eccentricity asteroids, an initial tether rotational speed is necessary, to make the tether rotate around the asteroid instead of oscillating about equilibrium.

Although very long tethers and massive ballast masses are necessary to obtain sufficient diversion, it is believed that attachment of multiple tethers and ballasts, with shorter tethers and lighter ballasts, would be able to do the same job. The precise verification of this fact will be considered in the future.

Simulation results have confirmed that by cutting the tether at an appropriate time, better diversion can be achieved in shorter time. By proper choice of the cutting point, it is possible to get about twice the diversion achieved without cutting the tether. A precise study to determine the characteristics of the optimal cut point needs to be done, which has been left as a future task.

In order to get enough diversion through tethering the asteroid to ballast mass, it is necessary to initiate the process well ahead of time. 15 years is a reasonable lead time to initiate tether assisted asteroid diversion.

<sup>1</sup> T.J. Ahrens and A.W. Harris, "Deflection and fragmentation of near-Earth asteroids." *Nature*. Vol. 360, No. 6403, 1992, pp. 429-433.

<sup>2</sup> H.J. Melosh and I.V. Nemchinov, "Solar asteroid diversion." *Nature*. Vol. 366, No. 6450, 1993, pp. 21-22.

<sup>3</sup> E.T. Lu and S.G. Love, "Gravitational tractor for towing asteroids." *Nature*. Vol. 438, No. 7065, 2005, pp. 177-178.

<sup>4</sup> C.R. McInnes, "Near Earth Object orbit modification using gravitational coupling." *Journal of Guidance Control and Dynamics*. Vol. 30, No. 3, 2007, pp. 870-873.

<sup>5</sup> D.B. French and A.P. Mazzoleni, " Asteroid diversion using a long tether and ballast " *Journal of Spacecraft and Rockets*. Vol. 46, No. 3, 2009, pp. 645-661.

<sup>6</sup> V. Chobotov and N. Melamed, "Deflection of Near Earth Objects by means of tethers." *Planetary Defense Conference*. Washington, D.C, 2007, pp. 2-7.